MATHEMATICS - CET 2024 - VERSION CODE – B-2
KEYS

1. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let z = 4x + 6y be the objective function. The minimum value of z occurs at

   (A) Only (0, 2)

   (B) Only (3, 0)

   (C) The mid-point of the line segment joining the points (0, 2) and (3, 0)

   (D) Any point of the line segment joining the points (0, 2) and (3, 0)

   Ans (D)

   $Z = 4x - 6y$

Corner points	Corresponding value Z
A (0, 2)	12
B (3, 0)	12
C (6, 0)	24
D (6, 8)	72
E (0, 5)	30

2. A die is thrown 10 times. The probability that an odd number will come up at least once is

   (A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$ (C) $\frac{1023}{1024}$ (D) $\frac{1}{1024}$

   Ans (C)

   $P(X = r) = {}^nC_r\, q^{n-r}\, p^r$

   $P = \frac{1}{2}$

   $q = \frac{1}{2}$

   $P(X \geq 1) = 1 - P(x < 1)$

   $= 1 - P(X = 0)$

   $= 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^{10}$

   $= 1 - \frac{1}{1024}$

   $= \frac{1023}{1024}$

Note:
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3. A random variable X has the following probability distribution:

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

   If the mean of the random variable X is $\frac{1}{3}$, then the variance is

   (A) $\frac{1}{18}$ (B) $\frac{5}{18}$ (C) $\frac{7}{18}$ (D) $\frac{11}{18}$

Ans (B)

$\frac{25}{36}+k+\frac{1}{36}=1$

$k=1-\frac{26}{36}=\frac{10}{36}$

$\mu=0+\frac{10}{36}+\frac{2}{36}=\frac{13}{36}=\frac{1}{3}$

$\text{Var}=0+\frac{10}{36}+\frac{4}{36}-\left(\frac{13}{36}\right)^2$

$=\frac{14}{36}-\frac{1}{9}$

$=\frac{14-4}{36}=\frac{10}{36}=\frac{5}{18}$

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4. If a random variable X follows the binomial distribution with parameters n = 5, p and P(X = 2) = 9P(X = 3), then p is equal to

(A) 10 (B) $\frac{1}{10}$ (C) 5 (D) $\frac{1}{5}$

Ans (B)

P(X = 2) = 9 P(X = 3)

q = 9p

1 − p = 9p

1 = 10p

$p=\frac{1}{10}$

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5. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

(A) 7, 6 (B) 5, 1 (C) 6, 3 (D) 8, 7

Ans (C)

$2^m = 56 + 2^n$

By inspection m = 6, n = 3

6. If $[x]^2 - 5[x] + 6 = 0$, where [x] denotes the greatest integer function, then

(A) $x \in [3, 4]$ (B) $x \in [2, 4)$ (C) $x \in [2, 3]$ (D) $x \in (2, 3]$

Ans (B)

t = [x]

$t^2 - 5t + 6 = 0$

(t − 2) (t − 3) = 0

t = 2 or t = 3

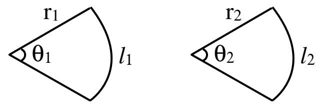
[x] = 2 [x] = 3

$x \in [2, 4)$

7. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is

(A) $\frac{5}{13}$ (B) $\frac{13}{5}$ (C) $\frac{13}{4}$ (D) $\frac{4}{13}$

Ans (B)



$l_1 = r_1\theta_1$ $\quad$ $l_2 = r_2\theta_2$

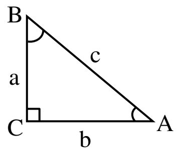
$l_1 = l_2 \Rightarrow r_1\theta_1 = r_2\theta_2$

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{78}{30} = \frac{26}{10} = \frac{13}{5}$$

8. If Δ ABC is right angled at C, then the value of tan A + tan B is

(A) a + b $\quad$ (B) $\frac{a^2}{bc}$ $\quad$ (C) $\frac{c^2}{ab}$ $\quad$ (D) $\frac{b^2}{ac}$

Ans (C)



Given, A + B = 90°

$$\tan A + \tan B = \frac{a}{b} + \frac{b}{a}$$
$$= \frac{a^2 + b^2}{ab}$$
$$= \frac{c^2}{ab}$$

9. The real value of 'α' for which $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is

(A) $(n+1)\frac{\pi}{2}, n \in \mathbb{N}$ $\quad$ (B) $(2n+1)\frac{\pi}{2}, n \in \mathbb{N}$ $\quad$ (C) $n\pi, n \in \mathbb{N}$ $\quad$ (D) $(2n-1)\frac{\pi}{2}, n \in \mathbb{N}$

Ans (C)

$$Z = \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$$
$$= \frac{(1 - i\sin\alpha)(1 - 2i\sin\alpha)}{1 + 4\sin^2\alpha}$$

Im (Z) = 0

$-2\sin\alpha - \sin\alpha = 0$

$\sin\alpha = 0$

$\alpha = n\pi, n \in Z$

10. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

(A) Breadth ≤ 15 cm $\quad$ (B) Breadth ≥ 15 cm $\quad$ (C) Length ≤ 15 cm $\quad$ (D) Length = 15 cm

Ans (B)

$l = 5b$

$2[l + b] \geq 180$

$l + b \geq 90$

$6b \geq 90$

$b \geq 15$

11. The value of $^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ is

(A) $^{50}C_4$ (B) $^{50}C_3$ (C) $^{50}C_2$ (D) $^{50}C_1$

Ans (A)

$^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4 \left[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{49}C_3 + {}^{49}C_4$

$= {}^{50}C_4$

12. In the expansion of $(1 + x)^n$

$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + n\frac{C_n}{C_{n-1}}$ is equal to

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n}{2}$ (C) $\frac{n+1}{2}$ (D) 3n (n +1)

Ans (A)

Put n = 2

$\frac{{}^2C_1}{{}^2C_0} + 2\frac{{}^2C_2}{{}^2C_1}$

$= \frac{2}{1} + 2\frac{1}{2}$

= 2 + 1 = 3

In option (A) $\frac{2(3)}{2} = 3$ or used $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

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13. If S_n stands for sum to n-terms of a G.P. with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is

(A) $r^n + 1$ (B) $\frac{1}{r^n+1}$ (C) $r^n - 1$ (D) $\frac{1}{r^n-1}$

Ans (B)

$S_n = \frac{a(r^n-1)}{r-1}$

$S_{2n} = \frac{a(r^{2n}-1)}{r-1}$

$\frac{S_n}{S_{2n}} = \frac{\frac{a(r^n-1)}{r-1}}{\frac{a((r^n)^2-1)}{r-1}} = \frac{r^n-1}{(r^n-1)(r^n+1)}$

$= \frac{1}{r^n+1}$

14. If A.M. and G.M. of roots of a quadratic equation are 5 and 4 respectively, then the quadratic equation is

(A) $x^2 - 10x - 16 = 0$ (B) $x^2 + 10x + 16 = 0$ (C) $x^2 + 10x - 16 = 0$ (D) $x^2 - 10x + 16 = 0$

Ans (D)

$\frac{a+b}{2} = 5$ $\sqrt{ab} = 4$

$a + b = 10 \qquad ab = 16$

$x^2 - (a + b)x + ab = 0$

$x^2 - 10x + 16 = 0$

15. The angle between the line x + y = 3 and the line joining the points (1, 1) and (–3, 4) is

(A) $\tan^{-1}(7)$ (B) $\tan^{-1}\left(-\frac{1}{7}\right)$ (C) $\tan^{-1}\left(\frac{1}{7}\right)$ (D) $\tan^{-1}\left(\frac{2}{7}\right)$

Ans (C)

$x + y = 3$

$y = -x + 3$

$y = mx + c$

$m_1 = -1$

(1, 1), (–3, 4)

$m_2 = \frac{4-1}{-3-1} = \frac{3}{-4}$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{-3}{4} + 1}{1 + (-1)\left(\frac{-3}{4}\right)}\right|$

$= \left|\frac{\frac{1}{4}}{\frac{7}{4}}\right|$

$\theta = \tan^{-1}\left(\frac{1}{7}\right)$

16. The equation of parabola whose focus is (6, 0) and directrix is x = – 6 is

(A) $y^2 = 24x$ (B) $y^2 = -24x$ (C) $x^2 = 24y$ (D) $x^2 = -24y$

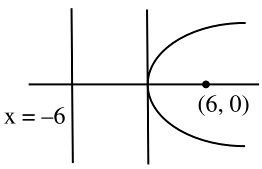
Ans (A)

$x = a = 6$

$y^2 = 4ax$

$y^2 = 4(6)\,x$

$y^2 = 24x$



17. $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to

(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Ans (C)

$\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = \frac{0}{0}$

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}(-\sin x)-0}{-\operatorname{cosec}^2 x}=\frac{-\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{-\left(\sqrt{2}\right)^2}$$
$$=\frac{1}{2}$$

18. The negation of the statement
"For every real number x; $x^2 + 5$ is positive"
is
(A) For every real number x; $x^2 + 5$ is not positive.
(B) For every real number x; $x^2 + 5$ is negative.
(C) Three exists at least one real number x such that $x^2 + 5$ is not positive.
(D) There exists at least one real number x such that $x^2 + 5$ is positive.
Ans (C)

19. Let a, b, c, d and e be the observations with mean m and standard deviation S. standard deviation of the observations a + k, b + k, c + k, d + k and e + k is
(A) kS (B) S + k (C) $\frac{S}{k}$ (D) S
Ans (D)

20. Let f : R → R be given by f(x) = tan x. Then f^{-1} (1) is
(A) $\frac{\pi}{4}$ (B) $\left\{n\pi+\frac{\pi}{4}:n\in Z\right\}$ (C) $\frac{\pi}{3}$ (D) $\left\{n\pi+\frac{\pi}{3}:n\in Z\right\}$
Ans (B)
$f^{-1}(1)=a$
$\Rightarrow 1 = f(a)$
$\Rightarrow 1 = \tan a$
$\Rightarrow\ a=n\pi+\frac{\pi}{4}n\in Z$

21. Let f : R → R be defined by f(x) = $x^2 + 1$. Then the pre images of 17 and –3 respective are
(A) ϕ, {4, – 4} (B) {3, –3}, ϕ (C) {4, –4}, ϕ (D) {4, –4}, {2, –2}
Ans (C)
$f(x)=17$
$x^2 + 1 = 17$
$x^2 = 16$
$x = \pm 4$
$f(x) = -3$
$x^2 + 1 = -3$
$x^2 = -4$
$x=\sqrt{-4}\notin R$

22. Let (gof) (x) = sin x and (fog) (x) = $\left(\sin\sqrt{x}\right)^2$. Then
(A) $f(x) = \sin^2 x, g(x) = x$ (B) $f(x)=\sin\sqrt{x},\ g(x)=\sqrt{x}$
(C) $f(x)=\sin^2 x,\ g(x)=\sqrt{x}$ (D) $f(x)=\sin\sqrt{x},\ g(x)=x^2$

Ans (C)

$g[f(x)] = \sin x$

$= \sqrt{\sin^2(x)}$

$f[g(x)] = (\sin\sqrt{x})^2$

$= \sin^2(\sqrt{x})$

23. Let A = {2, 3, 4, 5, …….. 16, 17, 18}. Let R be the relation on the set A of ordered pairs of positive integers defined by (a, b) R (c, d) if and only if ad = bc for all (a, b), (c, d) in A × A. Then the number of ordered pairs of the equivalence class of (3, 2) is

(A) 4 (B) 5 (C) 6 (D) 7

Ans (C)

Let (3, 2) R (x, y)

$\Rightarrow 3y = 2x \Rightarrow y = \frac{2x}{3}$

$\Rightarrow x = 3, y = 2$

$x = 6, y = 4$

$x = 9, y = 6$

$x = 12, y = 3$

$x = 15, y = 10$

$x = 18, y = 12$

24. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then x(y + z) + y (z + x) + z (x + y) equals to

(A) 0 (B) 1 (C) 6 (D) 12

Ans (C)

$\cos^{-1} x = \pi \Rightarrow x = -1$

$\cos^{-1} y = \pi \Rightarrow y = -1$

$\cos^{-1} z = \pi \Rightarrow z = -1$

$x(y + z) + y(z + x) + z(x + y)$

$= -1(-1 -1) -1 (-1 -1) -1(-1 -1)$

$= 6$

25. If $2 \sin^{-1} x - 3\cos^{-1} x = 4$, $x \in [-1, 1]$ then $2 \sin^{-1} x + 3\cos^{-1} x$ is equal to

(A) $\frac{4-6\pi}{5}$ (B) $\frac{6\pi-4}{5}$ (C) $\frac{3\pi}{2}$ (D) 0

Ans (B)

$2 \sin^{-1} x - 3 \cos^{-1} x = 4$

$2\left[\frac{\pi}{2} - \cos^{-1} x\right] - 3\cos^{-1} x = 4$

$\Rightarrow \pi - 2\cos^{-1} x - 3\cos^{-1} x = 4$

$\Rightarrow \pi - 5 \cos^{-1} x = 4 \quad \Rightarrow \cos^{-1} x = \frac{\pi - 4}{5}$

$\therefore\ 2 \sin^{-1}x + 3 \cos^{-1} x = 2 \sin^{-1} x + 2 \cos^{-1} x + \cos^{-1} x$

$= 2[\sin^{-1} x + \cos^{-1} x] + \cos^{-1} x$

$= 2\left[\frac{\pi}{2}\right] + \frac{\pi - 4}{5} = \frac{6\pi - 4}{5}$

26. If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ is equal to

(A) 7A – I (B) 7A (C) 7A + I (D) I – 7A

Ans (C)

$(I + A)^3 = (I + A)(I + A)(I + A)$

$= (I^2 + IA + AI + A^2)(I + A)$

$= (I + 2A + A^2)(I + A)$

$= (I + 3A)(I + A) \left[\because A^2 = A\right]$

$= I^2 + IA + 3AI + 3A^2$

$= I + 4A + 3A^2$

$= I + 7A \left[\because A^2 = A\right]$

27. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(A) 2^8 A (B) 2^9 A (C) 2^{10} A (D) 2^{11} A

Ans (B)

$A^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

$A^2 = 2^1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Also, $A^3 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

$A^3 = 2^2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\therefore \quad A^{10} = 2^9 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

28. If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then f(1) · f(3) + f(3) · f(5) + f(5) · f(1) is

(A) –1 (B) 0 (C) 1 (D) 2

Ans 1217160 **[None of the options Matches]**

$f(1) = -1610$

$f(3) = -756$

$f(5) = 0$

Grace

29. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(A) 4 (B) 5 (C) 11 (D) 0

Ans (C)

$|\text{Adj}\,A| = |A|^{n-1}$

$1(0) - \alpha(-2) + 3(-2) = (4)^2$

$2\alpha - 6 = 16$

$2\alpha = 22$

$\alpha = 11$

30. If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

(A) 3A (B) – 3B (C) 3B + 1 (D) 1 – 3A

Ans (A)

$A = x^2 - 1$

$B = x(x^2-1) - 1(x-1) + 1(1-x)$

$= x^3 - 3x + 2$

$\frac{dB}{dx} = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3A$

31. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x\to 0} \frac{f(x)}{x^2} =$

(A) – 1 (B) 0 (C) 3 (D) 2

Ans (B)

$f(x) = \cos x(-x^2) - x(0) + 1(x \sin x)$

$\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0}\left[-\cos x + \frac{\sin x}{x}\right]$

$= -1 + 1$

$= 0$

32. Which one of the following observations is correct for the features of logarithm function to any base $b > 1$?

(A) The domain of the logarithm function is R, the set of real numbers

(B) The range of the logarithm function is R^+, the set of all positive real numbers.

(C) The point (1, 0) is always on the graph of the logarithm function.

(D) The graph of the logarithm function is decreasing as we move from left to right.

Ans (C)

Note:
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33. The function $f(x) = |\cos x|$ is

(A) everywhere continuous and differentiable

(B) everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$

(C) neither continuous not differentiable at $(2n+1)\frac{\pi}{2}, n \in Z$

(D) not differentiable everywhere

Ans (B)

Standard result

34. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at x = 1 is

(A) 2 (B) 6 (C) 3 (D) 1

Ans (B)

$y = 2(x)^{3x}$

$\log y = \log 2 + 3x \log x$

$\frac{1}{y}\frac{dy}{dx} = 0 + 3(1) + 3\log x$

$\frac{dy}{dx} = y(3 + 3\log x)$

at $x = 1$

$\frac{dy}{dx} = (2)(3+0) = 6$

35. Let the function satisfy the equation f(x + y) = f(x) f(y) for all x, y ∈ R, where f(0) ≠ 0. If f(5) = 3 and $f'(0) = 2$, then $f'(5)$ is

(A) 6 (B) 0 (C) 5 (D) – 6

Ans (A)

$$f'(5) = \lim_{x\to 0}\frac{f(x)-f(5)}{x-5}$$

$$= \lim_{x\to 5}\frac{f(5+h)-f(5)}{h}$$

$$= \lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$$

$$= \lim_{h\to 0}\frac{f(5)[f(h)-1]}{h-0}$$

$$= \lim_{h\to 0} f(5)f'(0)$$

= (3) (2) = 6

36. The value of C in (0, 2) satisfying the mean value theorem for the function $f(x) = x(x-1)^2$, x ∈ [0, 2] is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans (B)

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$f(2) = 2(2-1)^2 = 2$

$f(0) = 0$

$f'(x) = 2x(x-1) + (x-1)^2$

$= 2x^2 - 2x + x^2 - 2x + 1$

$= 3x^2 - 4x + 1$

$f'(c) = 3c^2 - 4c + 1$

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$3c^2 - 4c + 1 = \frac{2-0}{2} = 1$

$3c^2 - 4c = 0$

$c = \frac{4}{3}$

37. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

(A) $-\frac{3}{4}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans (D)

Put x = 2 cos θ $\Rightarrow \cos\theta = \frac{x}{2} \Rightarrow \theta = \cos^{-1}\frac{x}{2}$

$\frac{d}{dx}\left[\cos^2\left[\cot^{-1}\sqrt{\frac{2+2\cos\theta}{2-2\cos\theta}}\right]\right]$

$= \frac{d}{dx}\left[\cos^2\left[\cot^{-1}\left(\cot\frac{\theta}{2}\right)\right]\right]$

$= \frac{d}{dx}\left[\cos^2\frac{\theta}{2}\right] = \frac{d}{dx}\left[\frac{1+\cos\theta}{2}\right]$

$= \frac{d}{dx}\left[\frac{1+\frac{x}{2}}{2}\right] = \frac{1}{4}$

38. For the function $f(x) = x^3 - 6x^2 + 12x - 3$; x = 2 is

(A) a point of minimum (B) a point of inflexion (C) not a critical point (D) a point of maximum

Ans (B)

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

$f'''(x) = 6$

$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 12x + 12 = 0$

$\Rightarrow \quad x^2 - 4x + 4 = 0$

$\Rightarrow \quad x = 2$

at x = 2, $f''(x) = 0$

$f'''(x) \neq 0$

$\therefore$ x = 2 is a point of inflection

39. The function x^x; x > 0 is strictly increasing at

(A) $\forall x \in \mathbb{R}$ (B) $x < \frac{1}{e}$ (C) $x > \frac{1}{e}$ (D) x < 0

Ans (C)

$y = f(x) = x^x$

log y = x log x

$\frac{1}{y}\frac{dy}{dx} = 1 + \log x$

$\frac{dy}{dx} = x^x(1 + \log x)$

For strictly increasing $\frac{dy}{dx} > 0$

$\Rightarrow (1 + \log x) > 0$

$\Rightarrow \log x > -1$

$x > e^{-1}$

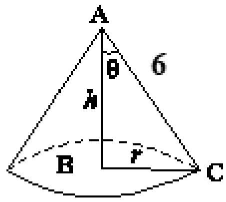
$x > \frac{1}{e}$

40. The maximum volume of the right circular cone with slant height 6 units is

(A) $4\sqrt{3}\pi$ cubic units (B) $16\sqrt{3}\pi$ cubic units (C) $3\sqrt{3}\pi$ cubic units (D) $6\sqrt{3}\pi$ cubic units

Ans (B)

Volume $V = \frac{1}{3}\pi r^2 h$



$h^2 + r^2 = 36 \Rightarrow r^2 = 36 - h^2$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}(36 - h^2)h = \frac{\pi}{3}(36h - h^3)$

$\frac{dV}{dh} = \frac{\pi}{3}(36 - 3h^2) = 0$

$3h^2 = 36$

$h^2 = 12$

$h = \sqrt{12}$

$r^2 = 36 - 12 = 24$

$\Rightarrow r = \sqrt{24}$

$\therefore V = \frac{\pi}{3}(24)2\sqrt{3} = 16\pi\sqrt{3}$

41. If $f(x) = x\,e^{x(1-x)}$ then f(x) is

(A) increasing in $\mathbb{R}$

(B) decreasing in $\mathbb{R}$

(C) decreasing in $\left[-\frac{1}{2}, 1\right]$

(D) increasing in $\left[-\frac{1}{2}, 1\right]$

Ans (D)

$f(x) = x\,e^{x(1-x)}$

$f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1-2x)$

$f'(x) = 0 \Rightarrow e^{x(1-x)}(1 + x(1-2x)) = 0$

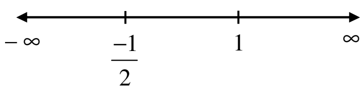
$\Rightarrow e^{x(1-x)}(1 + x - 2x^2) = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow 2x^2 - 2x + x - 1 = 0$

$\Rightarrow 2x(x - 1) + 1(x - 1) = 0$

$\Rightarrow x = 1,\ 2x = -1 \quad x = \frac{-1}{2}$



$x = 2 \qquad f'(2) = 2^{2(1-2)}(1 + 2 - 4) < 0$

$x = 0 \qquad f'(0) = e^{0(1-0)}(1 + 0 - 0) > 0$

$x = -1 \qquad f'(-1) = e^{-1}(1+1)(1-1-2) < 0$

f(x) is increasing on $\left[\frac{-1}{2}, 1\right]$

42. $\int \frac{\sin x}{3+4\cos^2 x}\,dx =$

(A) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$

(B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

(C) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

(D) $-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{3}\right)+C$

Ans (A)

$\int \frac{\sin x}{3+4\cos^2 x}dx$

cos x = t –sin x dx = dt

$I=\int\frac{-dt}{3+4t^2}=\frac{1}{4}\int\frac{dt}{\frac{3}{4}+t^2}$

$I=\frac{-1}{4}\int\frac{dt}{\left(\frac{\sqrt{3}}{2}\right)^2+t^2}=\frac{-1}{4}\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{t}{\frac{\sqrt{3}}{2}}\right)+C$

$I=\frac{2}{-4\sqrt{3}}\tan^{-1}\left(\frac{2t}{\sqrt{3}}\right)+C$

$I=\frac{1}{-2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$

43. $\int_{-\pi}^{\pi}(1-x^2)\sin x\cdot\cos^2 x\,dx =$

(A) $\pi-\frac{\pi^2}{3}$ (B) $2\pi-\pi^3$ (C) $\pi-\frac{\pi^3}{2}$ (D) 0

Ans (D)

$I=\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\,dx$

$f(x) = (1 - x^2)\sin x \cos^2 x$

$f(-x) = (1 - x^2)\sin(-x)\cos^2(-x)$

$= -(1 - x^2)\sin x\cos^2 x$

$= -f(x)$

$\therefore$ f(x) is odd function

$I=\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\,dx=0$

44. $\int\frac{1}{x\left[6(\log x)^2+7\log x+2\right]}dx =$

(A) $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$

(B) $\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$

(C) $\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

(D) $\frac{1}{2}\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

Ans (B)

$$I=\int\frac{1}{x\left[6(\log x)^2+7\log x+2\right]}dx$$

$$\log x=t\Rightarrow\frac{1}{x}dx=dt$$

$$I=\int\frac{dt}{(6t^2+7t+2)}$$

$$I=\int\frac{dt}{(6t^2+4t+3t+2)}$$

$$I=\int\frac{dt}{2t(3t+2)+1(3t+2)}$$

$$I=\int\frac{dt}{(3t+2)(2t+1)}$$

$$I=\int\frac{dt}{(3t+2)(2t+1)}=\frac{A}{(3t+2)}+\frac{B}{2t+1}$$

1 = A(2t + 1) +B (3t + 2)

$$t=\frac{-1}{2}\qquad 1=B\left(\frac{-3}{2}+2\right)\quad 1=B\left(\frac{1}{2}\right)$$

B = 2

$$t=\frac{-2}{3}\qquad 1=A\left(2\times-\frac{2}{3}+1\right)$$

$$1=A\left(-\frac{4}{3}+1\right)\Rightarrow 1=A\left(\frac{-1}{3}\right)$$

A = –3

$$I=\int\left(\frac{-3}{3t+2}+\frac{2}{2t+1}\right)dt$$

$$I=-3\frac{\log(3t+2)}{3}+2\frac{\log(2t+1)}{2}+C$$

I = log (2t + 1) – log(3t + 2) + C

$$I=\log\left(\frac{2t+1}{3t+2}\right)+C$$

$$I=\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$$

45. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx=$

(A) 2x + sin x + 2 sin 2x + C

(B) x + 2 sin x + 2 sin 2x + C

(C) x + 2 sin x + sin 2x + C

(D) 2x + sin x + sin 2x + C

Ans (C)

$$I=\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$$

$$I=\int\frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\,dx$$

$$I=\int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

$$I=I=\int\left(\frac{3\sin x-4\sin^3 x+2\sin x\cos x}{\sin x}\right)dx$$

$$I=\int(3-4\sin^2 x+2\cos x)dx$$

$$I=\int(3-2(1-\cos 2x)+2\cos x)dx$$

$$I=\int(1+2\cos 2x+2\cos x)dx$$

$I = x + \sin 2x + 2\sin x + C$

46. $\int_1^5 (|x-3|+|1-x|)dx =$

(A) 12 (B) $\frac{5}{6}$ (C) 21 (D) 10

Ans (A)

$$I=\int_1^5(|x-3|+|1-x|)dx$$

$$I=\int_1^3(3-x)dx+\int_3^5(x-3)dx+\int_1^5(x-1)dx$$

$$I=\left(3x-\frac{x^2}{2}\right)_1^3+\left(\frac{x^2}{2}-3x\right)_3^5+\left(\frac{x^2}{2}-x\right)_1^5$$

$$I=\left[\left(9-\frac{9}{2}\right)-\left(3-\frac{1}{2}\right)\right]+\left[\left(\frac{25}{2}-15\right)-\left(\frac{9}{2}-9\right)\right]+\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$$

$$I=\left[\frac{9}{2}-\frac{5}{2}\right]+\left[\frac{-5}{2}+\frac{9}{2}\right]+\left[\frac{15}{2}+\frac{1}{2}\right]$$

$$I=\frac{4}{2}+\frac{4}{2}+\frac{16}{2}=2+2+8=12$$

47. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+...+\frac{1}{5n}\right)=$

(A) $\frac{\pi}{4}$ (B) $\tan^{-1}3$ (C) $\tan^{-1}2$ (D) $\frac{\pi}{2}$

Ans (C)

$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+...\frac{1}{5n}\right]$$

$$=\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+...+\frac{n}{n^2+(2n)^2}\right]$$

$$=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

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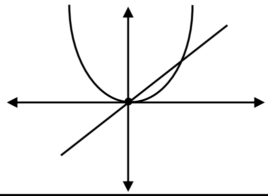
$$\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)^2}\right)$$

$$\int_0^2\frac{dx}{1+x^2}=\tan^{-1}x\Big|_0^2=\tan^{-1}2-\tan^{-1}0=\tan^{-1}2$$

48. The area of the region bounded by the line y = 3x and the curve $y = x^2$ in sq. units is

(A) 10 (B) $\frac{9}{2}$ (C) 9 (D) 5

Ans (B)



$y = 3x$ … (1)

$y = x^2$ … (2)

$3x = x^2 \Rightarrow x^2 - 3x = 0 \Rightarrow x(x-3) = 0$

$\Rightarrow x = 0 \quad x = 3$

Required area $=\int_0^3(3x-x^2)dx$

$=\left(\frac{3x^2}{2}-\frac{x^3}{3}\right)_0^3$

$=\left(\frac{3}{2}\times 9-\frac{27}{3}\right)$

$=27\left(\frac{1}{2}-\frac{1}{3}\right)$

$=27\left(\frac{1}{6}\right)=\frac{27}{6}=\frac{9}{2}$ sq units

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49. The area of the region bounded by the line y = x and the curve $y = x^3$ is

(A) 0.2 sq. units (B) 0.3 sq. units (C) 0.4 sq. units (D) 0.5 sq. units

Ans (D)

$y = x$ …(1)

$y = x^3$ … (2)

$x = x^3 \Rightarrow x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$

$\Rightarrow x = 0 \quad x = 1 \quad x = -1$

Required area $=2\int_0^1(x-x^3)dx$

$=2\left[\frac{x}{2}-\frac{x^4}{4}\right]_0^1$

$=2\left[\frac{1}{2}-\frac{1}{4}\right]$

$=2\left(\frac{4-2}{8}\right)=2\left(\frac{1}{4}\right)=\frac{1}{2}$

Note:
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50. The solution of $e^{\frac{dy}{dx}}=x+1,\ y(0)=3$ is

(A) y – 2 = x log x – x (B) y – x – 3 = x log x

(C) y – x – 3 = (x + 1) log (x + 1) (D) y + x – 3 = (x + 1) log (x + 1)

Ans (D)

$e^{\frac{dy}{dx}} = x + 1,\ y(0) = 3$

$\frac{dy}{dx} = \log(x+1)$

dy = log(x+1) dx

Integrating on both side

$\int dy = \int \log(x+1)dx$

$y = x\log(x+1) - \int\left(\frac{x}{x+1}\right)dx$

$y = x\log(x+1) - \int\left(\frac{x+1-1}{x+1}\right)dx$

y = xlog(x + 1) – x +log(x + 1) +C

when x = 0 y = 3 we get

3 = 0 log(0+1) – 0 + log (0+1) +C

C = 3

Here, y = x log (x+1) – x + log(x + 1) + 3

y + x – 3 = (x +1) log (x + 1)

51. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

(A) xy = C (B) $x^2 + y^2 = C$ (C) $x^2 - y^2 = C$ (D) $\frac{y}{x} = C$

Ans (A)

Equation of tangent will be

$\frac{x}{2h} + \frac{y}{2k} = 1$

$\frac{y}{2k} = 1 - \frac{x}{2h}$

$\Rightarrow y = -\left(\frac{2k}{2h}\right)x + 2k$

Slope = $-\frac{k}{h}$

∴ Family of curves at a point (h, k) on the curve, the slope of tangent is $-\frac{k}{h}$

$\frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

$\int \frac{1}{y}dx = -\int \frac{1}{x}dx$

⇒ log y + log x = log c

∴ xy = c

52. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a ΔABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

Ans (C)

Let A be the origin of reference so that the position vector of

$\overrightarrow{AB}=$ position vector of B

$\overrightarrow{AC}=$ position vector of C

$\therefore$ position vector of midpoint of B and C

$\vec{P}=\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$

$=4\hat{i}-\hat{j}+4\hat{k}$

Length $=|\overrightarrow{AP}|=\sqrt{33}$

53. The volume of the parallelopiped whose co-terminous edges are $\hat{j}+\hat{k},\ \hat{i}+\hat{k}$ and $\hat{i}+\hat{j}$ is

(A) 6 cu. units (B) 2 cu. units (C) 4 cu. units (D) 3 cu. units

Ans (B)

Volume $=\begin{vmatrix}0&1&1\\1&0&1\\1&1&0\end{vmatrix}$

$\therefore$ V = 0(0 − 1) − 1 (0 − 1) + 1 (1 − 0)

= 0 + 1 + 1

Volume = 2 cu. units

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54. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\dfrac{\pi}{4}$ (B) $\theta=\dfrac{\pi}{3}$ (C) $\theta=\dfrac{2\pi}{3}$ (D) $\theta=\dfrac{\pi}{2}$

Ans (C)

If $\vec{a}$ and $\vec{b}$ are unit vectors then

$\cos\dfrac{\theta}{2}=\dfrac{|\vec{a}+\vec{b}|}{2}$

$=\dfrac{1}{2}$

$\dfrac{\theta}{2}=60°\rightarrow\theta=120°$

55. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors and p, q, r are vectors defined by

$\vec{p}=\dfrac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]},\ \vec{q}=\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]},\ \vec{r}=\dfrac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, then $(\vec{a}\times\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Ans (D)

Given that $\vec{p}=\dfrac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]},\vec{q}=\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$$\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}$$

$$= (\vec{a}+\vec{b})\cdot\vec{p} + (\vec{b}+\vec{c})\cdot\vec{q} + (\vec{c}+\vec{a})\cdot\vec{r}$$

$$= \vec{a}\cdot\vec{p} + \vec{b}\cdot\vec{p} + \vec{b}\cdot\vec{q} + \vec{c}\cdot\vec{q} + \vec{c}\cdot\vec{r} + \vec{a}\cdot\vec{r}$$

$$= \frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]} + \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]} + \frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}$$

= 1 + 1 + 1

= 3

> **Note:**
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56. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

(A) $-\frac{10}{7}$ (B) $-\frac{7}{10}$ (C) −10 (D) −7

Ans (A)

$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$

If two lines are perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(-3)(3k) + (2k)(1) + (2)(-5) = 0$

$-9k + 2k - 10 = 0$

$-7k = 10 \quad \therefore k = -\frac{10}{7}$

57. The distance between the two planes 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is

(A) 2 units (B) 8 units (C) $\frac{2}{\sqrt{29}}$ units (D) 4 units

Ans (C)

2x + 3y + 4z = 4 … (1)

4x + 6y + 8z = 12

⇒ 2x + 3y + 4z = 6 … (2)

Given, two lines are parallel

$$\therefore D = \left|\frac{d_1 - d_2}{\sqrt{a^2+b^2+c^2}}\right| = \left|\frac{6-4}{\sqrt{2^2+3^2+4^2}}\right| = \left|\frac{2}{\sqrt{4+9+16}}\right| = \frac{2}{\sqrt{29}} \text{ units}$$

> **Note:**
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58. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and the plane 2x − 2y + z = 5 is

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$ (C) $\frac{3}{50}$ (D) $\frac{3}{\sqrt{50}}$

Ans (A)

$\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and 2x − 2y + z = 5

From equation of line, the direction vector $\vec{s} = (l, m, n) = (3, 4, 5)$

From equation of plane, the normal vector $\vec{q} = (A, B, C) = (2, -2, 1)$

$$\therefore \sin\theta = \frac{|Al + Bm + Cn|}{\sqrt{A^2+B^2+C^2}\sqrt{l^2+m^2+n^2}}$$

$$\sin\theta = \left|\frac{6-8+5}{\sqrt{4+4+1}\sqrt{9+16+25}}\right|$$

$$\Rightarrow \sin\theta = \frac{3}{3\sqrt{50}} = \frac{1}{5\sqrt{2}}$$

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59. The equation xy = 0 in three-dimensional space represents
   (A) a pair of straight lines
   (B) a plane
   (C) a pair of planes at right angles
   (D) a pair of parallel planes

   Ans (C)

   The equation xy = 0 can be written as y = 0 or x = 0. There are two separate equations that represent two planes in 3–D.

   The 2 planes represented by the equation xy = 0 are perpendicular to each other. This can be seen by analyzing the equation in term of 3 coordinates x, y, z.

   The equation xy = 0 implies that either x = 0 or y = 0.

   This means that the 2 planes are parallel to the z-axis, and then intersect it at right angle.

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60. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is
   (A) x − y + z = 1
   (B) x + y + z = 5
   (C) x + 2y − z = 1
   (D) 2x − y + z = 5

   Ans (A)

   dr's of line : 1, 5, 4

   dr's of normal to the plane

   (A) 1, –1, 1

   (B) 1, 1, 1

   (C) 1, 2, –1

   (D) 2, –1, 1

   Plane contains line

   ⇒ line is perpendicular to normal

   By observation (A) satisfies

Note:
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